Variance-Reducing Incentives for Labor Contracts

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Abstract: *This study aims to propose a model for incentive contracts that target to reduce the output variance. It is a general type of various models suggested in the literature in this framework. The most important contribution of the proposed model is that a variety of observed contracts, for instance bonus plans and stock options can be derived from it by varying the assumptions about the observability of the variance-reducing actions and about the agent's degree of risk aversion. The conclusions suggest that one should not disregard the relevance of variance-reducing actions because disregarding them misleads us about the characteristics of the optimal contract and an inefficient choice of methods to handle moral hazard problem.*

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I. Introduction

Monitoring is a practical tool to control the moral hazard situations. The value of monitoring will increase with a fall in the variance of this information. This study suggests a model under which variance-reduction can be attained under different informational settings. In a principal-agent framework, a fall in outcome variance improves the principal's capacity to utilize the outcome to observe agent's effort. The standard principal-agent literature generally focuses on the mean-increasing effort and leaving uncertainty present attached to mean of the outcome (i.e. Shavel, 1979; Grossman and Hart, 1983a; Grossman and Hart, 1983b; Mirrlees, 1999; Chade and Serio, 2002). However, the treatment of outcome variance lessens unobservability and, thus, brings more certainty to the model under consideration yet such a treatment is not without a cost. The first condition for introducing such a motivation is that the principal should pay the costs of variance-reducing effort to the agent. Moreover, a moral hazard problem takes place whenever the variance-reducing effort is unobservable. In other words, this study is an attempt to resolve the optimality of contracts with variance-reducing incentives.

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In the literature, there are various studies on incentive contracts towards reduction of variance in specific areas of interest (i.e. Milgrom, 1981; Bester, 1985; Healy 1985; Baker, 1992; Holmstrom and Milgrom, 1994; Melumad et al., 1995; Al-Najjar, 1997; Dewatripont et al., 2000; Faynzilberg and Kumar, 2000; Lundesgaard, 2001; Agrawal, 2002; Squintani, 2003; etc.). However, our study displays a complete approach of what are investigated in those studies. In other words, it offers a more general model that can be employed under different informational environments and for different types of incentive contracts.

II. The Model and the Assumptions

In the model presented here, it is assumed that the principal is risk-neutral and the agent is risk averse. The product market consists of identical firms producing homogenous products and labor is the only variable factor of production. The agent is potentially capable of two types of effort or action; the first one affects the mean of the outcome, e_1, and the second one affects the variance of the outcome, e_2. An increase in e_1 increases the mean by shifting the outcome distribution to the right in the sense of first-order stochastic dominance. An increase in e_2 decreases the outcome variance and concentrates the outcome distribution around the mean in the sense of second-order stochastic dominance. The agent is assumed to have an additive utility function, W,

(1) $$W(I, e_1, e_2) = U(I) - V(e_1, e_2)$$

where I stands for income, $\partial W/\partial I > 0$ and $\partial W/ \partial e_i, < 0, i=1,2$.

Further, it is also assumed that $U(I)$ is a power utility function,

(2) $$U(I) = \frac{I^{1-\alpha}}{1-\alpha} \quad , 0 < \alpha < 1$$

Here α is the agent's coefficient of relative risk aversion. The outcome of the contract is output which is denoted by x and it is assumed to be normally distributed with mean μ and variance σ^2.

(3) $\mu = \mu(e_1) \quad , d\mu(e_1)/de_1 > 0$

$$\sigma^2 = \sigma^2(e_2) \quad , d\sigma^2(e_2)/de_2 < 0$$

Assuming normal distribution simplifies the analysis by separating the effects of the mean-increasing action from the variance-decreasing action.

The principal's problem, on the other hand, is to maximize the expected utility, but he should reach this maximized utility subject to the agent's optimization problem, agent's reservation utility, $\overline{U}$. In fact, the principal is constrained by the labor market conditions. Moreover, he also pays a minimum wage which indicates that penalties imposed on the agent are bounded from below. This is the limited liability constraint. Lastly, the principal is constrained by the agent's utility-maximizing choice of his unobservable actions; this is the incentive compatibility constraint. It is further assumed that there exists a solution to the principal's problem and e_1^* is strictly positive. On the other hand, the optimal variance-reducing action, e_2^*, is weakly positive.

Let $Z = E\{W(I, e_1, e_2)\}$ and $Z_i = \partial Z / \partial e_i$, $i = 1,2$. Under these assumptions, the principal's problem is

(4) $\underset{x}{Max.} E\{x - I(x)\}$

such that

$$E\{W(I, e_1, e_2)\} \geq \overline{U}$$

$$I(x) \geq 0$$

$$Z_2 = 0, \text{ or } e_2^* = 0 \text{ if } Z_2 < 0 \ \forall \ e_2 > 0.$$

The first constraint ensures the participation of the agent. The second constraint is the limited liability constraint and the last two constraints are incentive compatibility constraints.

Let $f(x)$ be the density function of x, and $f'(x) = \partial f(x)/\partial e_1$. As Holmstrom (1979) specifies that f'/f is the derivative of the maximum likelihood function $\log f$ when e_1 is viewed as an unknown parameter. f'/f exhibits the magnitude of agent's deviation from e_1^*, and the sign of it shows the direction of this deviation. Therefore, this ratio can be evaluated as a measure of the informativeness of the outcome about the agent's productive action. Thus, if e_1 is observable the principle derives no benefits as $e_2 > 0$, whereas incurs the cost of compensating the agent for his variance-reducing action. In conclusion, the principal's optimal choice is $e_2^* = 0$. Given the optimal compensation rule, $I^*(x)$, any variance-reducing actions only increases the agent's disutility from effort without reducing his risk. Thus, agent also chooses the $e_2 = 0$. Then, it can be stated that the informative role of e_2 will be canceled out.

III. Optimal Results under Different Informational Environments

In this section of the study, the optimization problem will be put forward under different information environments. First of the cases assumes that mean-effecting effort, is observable but variance-reducing action is not observable. Then, the problem can be stated as

Proposition:[1]

(i) $$I^*(x) = \begin{cases} k\ , if\ e_1 = e_1^* \\ 0\ , otherwise \end{cases}$$

where k is constant, $U(k) = \overline{U} + V(e_1^*, e_2^*)$

(ii) $e_2^* = 0$

[1] *See the appendix for the proof of this proposition.*

(iii) $e_2^* = 0$ is the first best solution to the principal's problem.

With a fixed level of incentive payment, in other words independent of output produced, any variance-reducing action only increase the agent's disutility from effort leaving him at the same level of risk. Thus, it is also optimal for the agent to choose $e_2 = 0$.

The second case treats the situation where the mean-effecting action is unobservable but the variance-effecting action is observable. Then, the principal's problem can be written as

(5) $\quad Max. E\{x - I(x)\}$

such that

$$E\{W(I, e_1, e_2)\} \geq \overline{U}$$

$$I(x) \geq 0$$

$$Z_1 = 0 \ .$$

Let λ_1 and λ_2 be the associated Lagrange multipliers for the above first and the last constraints respectively. Then, the optimal incentive scheme will be

$$I^*(x) = \begin{cases} \{\lambda_1 + \lambda_2(\mu_1/\sigma^2)(x-\mu)\}^{1/\alpha}, & \textit{if } e_2 = e_2^* \textit{ and } \lambda_1 + \lambda_2(\mu_1/\sigma^2)(x-\mu) > 0 \\ 0, & \textit{otherwise} \end{cases}$$

where $\lambda_1 > 0$, $\lambda_2 > 0$ and if $V = 0$ and $V_{12} = 0$, while $e_2 = 0$ then $e_2^* > 0$.

Now, the principal imposes a binding contract including e_2^* by the observability of e_2, otherwise the principal will pay the minimum wage.

In the final case, both types of actions are assumed to be unobservable. Agent, now, is in a position to choose both types of actions in order to maximize his expected utility. For this case, the principal's problem can be stated as

(6) $\quad Max. E\{x - I(x)\}$

such that

$E\{W(I,e_1,e_2)\} \geq \overline{U}$

$I(x) \geq 0$

$Z_1 = 0$

$Z_2 = 0$

Let λ_1, λ_2, and λ_3 be the associated Lagrange multipliers for the above first, third and the last constraints respectively. The optimal incentive scheme can be written as

$$I^*(x) = \begin{cases} \left[\{\lambda_1 + \lambda_2(\mu_1/\sigma^2)(x-\mu)\} + \lambda_3\sigma_2^2/2\sigma_2\{((x-\mu)/\sigma))^2 - 1\}\right]^{1/\alpha} & \text{if} \quad C > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $C = \{\lambda_1 + \lambda_2(\mu_1/\sigma^2)(x-\mu)\} + \lambda_3\sigma_2^2/2\sigma_2\{((x-\mu)/\sigma))^2 - 1\}$, $\lambda_1 > 0$, $\lambda_2 > 0$;

if $0 < \alpha \leq 1/2$, then $\lambda_3 > 0$; if $1/2 < \alpha < 1$ and $\mu \to \infty$, then $\lambda_3 \geq 0$; if $V = 0$ and $V_{12} = 0$, then $e_2^* > 0$.

This type of contract will be subject to the aggregation of signals about the chosen levels of e_1 and e_2. Moreover, it is a close approximation to the real world situation. $(x - \mu)$ component can be evaluated as a signal of the agent's mean-increasing action. In other words, this deviation is a measure of the likelihood that $\mu(e_1) \neq \mu(e_1^*)$. As can be seen from the above equation, the share of the component of (x - μ) in the incentive scheme is (μ_1/σ^2) where μ_1 is the sensitivity of the expected outcome to the agent's mean-increasing action and the more sensitive the expected outcome the better the outcome as a signal about e_1.[2] On the other hand, σ^2 shows the noisiness of the signal, in other words, the higher the variance, the less precise is

$x - \mu(e_1)$. Thus, the weight given to $(x - \mu)$ is sensitivity to noise ratio to this signal. The deviation $\{((x - \mu)/\sigma))^2$ from its expected value of one shows the likelihood that

[2] The subscritps for μ and σ is decscribed with reference to e_1 and e_2.

$\sigma^2(e_2) \neq \sigma^2(e_2^*)$ and the share given to this signal in the incentive scheme is $\sigma_2^2/2\sigma_2$. Consequently, the role of these two weights in monitoring is extremely important.

The signs of Lagrange multipliers imply the direction in which the contract stimulates both type of efforts. The strictly positive multiplier λ_2 indicates the fact that agent's monetary compensation increases with the mean indicator, $(x - \mu)$, and provokes the productive effort. On the other hand, the strictly positive multiplier λ_3 shows that monetary compensation decreases with the variance indicator, $\{((x - \mu)/\sigma))^2 -1$, this encourages variance reduction, whereas strictly negative λ_3 implies that more risk-taking behavior is appropriate. It is important to note that variance-effecting activity is the result of the agent's degree of risk aversion. As the relative degree of risk aversion increases, the speed of variance reduction in outcome increases. However, one can insist upon the fact that the reduction in outcome variance has conflicting effects on the agent's expected utility. First, a negative effect can be observed depending upon the required effort. Second, a positive effect can be the result due to the lower compensation variance as income depends on the outcome. The second effect is relatively weaker as compared to the first one. Needless to say stronger incentive scheme is needed for e_2, the less risk averse the agent is. Consequently, if $0 < \alpha \leq 1/2$, then $\lambda_3 > 0$. For the more risk-averse agent, the contract with $\lambda_3 > 0$ may cause him to invest more than the needed level for e_2, and less than the level needed for e_1. Hence, if $1/2 < \alpha < 1$, then λ_3 may be zero.

In general, most of the compensation schemes increase or at least do not decrease in income since agent has ability to manipulate outcome before it is observed by the principal. The agent can guarantee a compensation scheme that is nondecreasing over the entire range of x by influencing the outcome. Agent reports the value of outcome that maximizes his compensation over the interval $[0,x]$. On the other hand, the principal's optimal policy is to present a nondecreasing contract. The agent is able to avoid any range of x where I declines by manipulating the outcome, thus, a nondecreasing contract offers the same incentives for variance-reducing contract but no outcome is exhausted. Therefore, the principal's problem can be written as:

(7) $Max.E\{x - I(x)\}$

such that

$$E\{W(I, e_1, e_2)\} \geq \overline{U}$$

$$I(x) \geq 0$$

$$Z_1 = 0.$$

$$Z_2 = 0, \text{ or } e_2^* = 0 \text{ if } Z_2 < 0 \ \forall \ e_2 > 0$$

$$\frac{\partial I(x)}{\partial x} = I'(x) \geq 0.$$

Let λ_1, λ_2, and λ_3 be the associated Lagrange multipliers for the above first, third and the fourth constraints respectively. The optimal incentive scheme can be written as

$$I^*(x) = \begin{cases} \left[\{\lambda_1 + \lambda_2(\mu_1/\sigma^2)(x-\mu)\} + \lambda_3\sigma_2^2/2\sigma_2\{((x-\mu)/\sigma))^2 - 1\}\right]^{1/\alpha} \textit{if} \quad C > 0 \quad \textit{and} \quad \dfrac{\partial C}{\partial x} > 0 \\ \left[\{\lambda_1 + \lambda_2(\mu_1/\sigma^2)(x-\mu)\} + \lambda_3\sigma_2^2/2\sigma_2\{((x-\mu)/\sigma))^2 - 1\}x_b\right]^{1/\alpha} \textit{if} \quad C(x_b) > 0 \quad \textit{and} \quad \dfrac{\partial C}{\partial x} \leq 0 \\ 0, \qquad \textit{otherwise} \end{cases}$$

where $C = \{\lambda_1 + \lambda_2(\mu_1/\sigma^2)(x-\mu)\} + \lambda_3\sigma_2^2/2\sigma_2\{((x-\mu)/\sigma))^2 - 1\}$,

$x_b = (x \mid \partial C/\partial x = 0)$, $\lambda_1 > 0$, $\lambda_2 > 0$; if $0 < \alpha \leq 1/2$, then $\lambda_3 > 0$; if $V = 0$ and $V_{12} = 0$, then $e_2^* > 0$.

The optimal compensation scheme, $I^*(x)$, presents the optimal incentive for variance reduction and avoids any output manipulation from the agent. If the agent is more risk averse ($1/2<\alpha<1$), he may require incentives to take risks which are profitable from the point of view of principle. Hence, the optimal contract may be increasing in x and convex with $\lambda_2 \leq 0$.

It should be noted that $\lambda_2 < 0$ might be optimal even if $\mu(e_1)$ is not bounded below because the agent never chooses $e_1=0$ if $I(x)$ is nondecreasing. The model presented, having those properties above, is consistent with contracts including bonus plans and stock options.

IV. Concluding Remarks

This paper proposes a model in which agent has ability to influence both the expected outcome and the variance of the outcome. Under situations in which agent's mean-increasing actions are unobservable, variance reduction is a desired property of the model. This property is also valid for a risk-neutral principal. Variance reduction enhances the informativeness of the outcome about the agent's mean-increasing actions. Therefore, it provides a tool for handling the moral hazard problem in labor markets. Nevertheless, the variance-reducing effort necessitates incentives if it is not observable. Therefore, it produces a moral hazard problem of its own. The motivation for variance-reducing effort is dependent upon the agent's level of risk aversion and the likelihood optimal contracts. Agents with low levels of risk aversion demand more incentives for variance-reducing effort while highly risk-averse agents require more incentives for mean-increasing effort. The most important contribution of our model is that a variety of observed contracts, for instance bonus plans and stock options can be derived from it by varying the assumptions about the observability of the variance-reducing actions and about the agent's degree of risk aversion. Previous attempts to model those cases legitimize each of these contracts, our model proposes a setting in which coherent with all of them.

The conclusions reached suggest that one should not disregard the relevance of variance-reducing actions since the ignorance of them will direct the researcher to draw irrelevant conclusions on the characteristics of the optimal contract and an inefficient choice of methods to overcome moral hazard problem.

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Appendix

Proof of Proposition 1: In order to prove this proposition, we utilize three lemmas.

Lemma 1: $\int_0^\infty (u^{k+1} - \lambda u^k) f(u) du = a^2 k \int_0^\infty u^{k-1} f(u) du, \quad \forall k > 0$

where $a = \lambda_1 \frac{\mu_1}{\sigma}$.

Proof: Using the formula

$$\int_0^\infty (u^{v-1} e^{-\phi u^2 - \xi u} du = (2\phi)^{-v/2} \Omega(v) \exp\left(\frac{\xi^2}{8\phi}\right) \Lambda_{-v}\left(\frac{\xi}{\sqrt{2\phi}}\right), \quad \phi > 0, v > 0,$$

where Λ is a parabolic cylinder function, and

$$\Lambda_{v+1}(n) - n\Lambda_v(n) + v\Lambda_{v-1}(n) = 0$$

It can be proved that

$$\int_0^\infty (u^{k+1} - \lambda u^k) f(u)du = \left[\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\lambda^2}{2a^2}\right) \exp\left(\frac{\lambda^2}{4a^2}\right) a^{k+2}\Omega(k+1)\right]\left[(k+1)\Lambda_{-k-2}\left(-\frac{\lambda}{a}\right) - \frac{\lambda}{a}\Lambda_{-k-1}\left(-\frac{\lambda}{a}\right)\right]$$

$$= a^2 k \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\lambda^2}{a^2}\right) \exp\left(\frac{\lambda^2}{4a^2}\right) a^k \Omega(k) \Lambda_{-r}\left(-\frac{\lambda}{a}\right)$$

$$= a^2 k \int_0^\infty u^{k-1} f(u)du \qquad \therefore$$

Lemma 2: If e_1 is unobservable, three possible cases for e_2 can be written as

(1) e_2 is unobservable and $e_2^* > 0$

(i) $I(x)$ is continuous and nondecreasing in x

$$I^*(x) = \begin{cases} [h(y)]^{\frac{1}{\alpha}} & if\ h(y) > 0\ and\ h'(y) > 0 \\ \left[h\left(-\frac{a}{b}\right)\right]^{\frac{1}{\alpha}} & if\ h\left(-\frac{a}{b}\right) > 0\ and\ h'(y) \le 0 \\ 0 & otherwise \end{cases}$$

(ii) $I(x)$ decrease in x

$$I^*(x) = \begin{cases} [h(y)]^{\frac{1}{\alpha}} & if\ h(y) > 0 \\ 0 & otherwise \end{cases}$$

where $y = \dfrac{x-\mu}{\sigma}$, $h(y) = \lambda + ay + \dfrac{1}{2}b(y^2 - 1)$, and $b = \lambda_2 \dfrac{\sigma_2^2}{\sigma^2}$

(2) e_2 is unobservable and $e_2^* = 0$

$$I^*(x) = \begin{cases} [u(e_2 = 0)]^{\frac{1}{\alpha}} & if\ u > 0 \\ 0 & otherwise \end{cases}$$

(3) e_2 is observable

$$I^*(x) = \begin{cases} [u(y)]^{\frac{1}{\alpha}} & if\ u(y) > 0\ and\ e_2 = e_2^* \\ 0 & otherwise \end{cases}$$

where $u(y) = \lambda + ay$

Proof: Let *κ(x)* and *τ(x)* be the Lagrange multipliers for the constraints $I(x) \geq 0$ and $I'(x) \geq 0$.

(1) The principal's problem in (7) can be stated as

$$\mathcal{L} = \int_{-\infty}^{\infty} (x - I) f(x) dx + \lambda_1 \left[\int_{-\infty}^{\infty} U(I) f(x) dx - V(e_1, e_2) - \overline{U} \right] + \lambda_2 \left[\int_{-\infty}^{\infty} U(I) f_1 dx - V_1 \right]$$

$$+ \lambda_3 \left[\int_{-\infty}^{\infty} U(I) f_2 dx - V_2 \right] + \kappa(x) I(x) + \tau(x) I'(x)$$

Maximizing $\mathcal{L}$ with respect to x and rearranging gives

$$\frac{\partial \mathcal{L}}{\partial I} - \frac{\partial}{\partial x}\left(\frac{\partial \mathcal{L}}{\partial I'}\right) = -f(x) + \lambda_1 U'(I) f(x) + U'(I)\left[\lambda_2 f_1(x) + \lambda_3 f_2(x)\right] + \kappa(x) + \tau'(x)$$

provided that $\kappa(x) > 0, \quad I^*(x) = 0.$

If $\kappa(x) = 0, \quad I^*(x) > 0$, then

$$I^*(x) = h^{\frac{1}{\alpha}} \left[\frac{1}{1 + \frac{\tau'(x)}{f(x)}} \right]^{\frac{1}{\alpha}}$$

where $\tau(x) > 0, \quad I'(x) > 0.$

(2) Under this situation, the last constraint in (6) and in (7) is $e_2^* = 0$. By substituting this constraint into (7),

$$\mathcal{L} = \int_{-\infty}^{\infty} (x - I(x)) f(x \mid e_2 = 0) dx + \lambda_1 \left[\int_{-\infty}^{\infty} U(I) f(x \mid e_2 = 0) dx - V(e_1, e_2 = 0) - \overline{U} \right]$$

$$+ \lambda_2 \left[\int_{-\infty}^{\infty} U(I) f_1(x \mid e_2 = 0) dx - V_1 \right] + \kappa(x) I(x) + \tau(x) I'(x)$$

Maximizing $\mathcal{L}$ with respect to *I* gives

$$I^*(x)=\begin{cases}[u(e_2=0)]^{\frac{1}{\alpha}}\left[\dfrac{1}{1+\dfrac{\tau'(x)}{f(x)}}\right]^{\frac{1}{\alpha}} & \text{if } u>0\\ 0 & \text{otherwise}\end{cases}$$

If $u>0$, $I'(x)>0, I^*(x)=u^{\frac{1}{\alpha}}$; $I^*(x)=0$ otherwise. For any interval, $I'(x)=0$, $I(x)$ is constant in x. The sign of λ_2 is same as the sign of $\dfrac{\partial u^{\frac{1}{\alpha}}}{\partial x}$. Therefore, if $\lambda_2\le 0$, $I^*(x)$ should be constant for all x. However, the agent chooses $e_1=0$. Thus, λ_2 should be strictly positive and $u^{\frac{1}{\alpha}}$ is increasing for all x, where $\tau'(x)=0,\forall x$.

(3) The incentive compatibility constraint with respect to e_2 is not binding because e_2 is observed. Hence, the term for λ_3 is cancelled off from the $\mathcal{L}$ in part (1). The rest of the proof is same as the proof in part (2). However, now, $u^{\frac{1}{\alpha}}$ is substituted for $[u(e_2=0)]^{\frac{1}{\alpha}}$ for the optimal incentive scheme. ∴

Lemma 3: λ_1 is strictly positive.

Proof: The lemma is proved for the case where e_2 is unobservable, $e_2^*>0$, and $I(x)$ is nondecreasing in x. Other proofs for different cases are similar.

$$\lambda=E\left[\frac{1}{U'(I)}\right]-\int_{-\infty}^{\infty}\frac{\kappa(x)}{U'(I)}dx+\int_{-\infty}^{\infty}\frac{\tau'(x)}{U'(I)}dx$$

Whenever $I(x)>0$, $\kappa(x)=0$. If $\kappa(x)$ is greater than zero, this means that $I^*(x)=0$. Thus, we have

$\int_{-\infty}^{\infty}\dfrac{\kappa(x)}{U'(I)}dx\to 0$. If $I'(x)=0$, then $\tau(x)=0$ that implies $\tau'(x)=0$. Let $\lfloor x_{1j},x_{2j}\rfloor$ be intervals of x where $j=1,2,\ldots N$. Moreover, assume that in those intervals $\tau'(x)\neq 0$ which further entails that $I'(x)=0$ and $I(x)$ is constant in *x*. Let $d_j(x)$ be the value of $I(x)$ for $x \in \lfloor x_{1j}, x_{2j} \rfloor$. Then, it can be written that

$$\lambda \to E\left[\frac{1}{U'(I)}\right] + \int_{-\infty}^{\infty} \frac{\tau'(x)}{U'(I)} dx = \int_{-\infty}^{x_{11}} \frac{1}{U'(I)} f(x)dx + \int_{x_{2N}}^{\infty} \frac{1}{U'(I)} f(x)dx$$

$$+ \sum_j \left[\int_{x_{2j}}^{x_{1j+1}} \frac{1}{U'(I)} f(x)dx + \frac{1}{U'd_j} \int_{x_{1j}}^{x_{2j}} \left(f(x) + \tau'(x)\right)dx \right]$$

When $I^*(x)=0$, $\frac{1}{U'(I)}$ approaches to zero. If $I^*(x)>0$, $\frac{1}{U'(I)}$ will be positive meaning that $f(x)+\tau'(x)$ will also be positive. Therefore, the last equation presents that λ will be strictly positive. $\therefore$